When Will the Conformal Flatness and Quasi-Equilibrium Assumptions Be Valid in NS Inspiral?

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Inspiral Approximations

• Binary neutron star systems are complex two body problems in general relativity

• Various approximations are used to study the different stages of the inspiral
  – at separations much larger than the neutron star radius post-Newtonian approximations are sufficient
  – at separations of a few radii, hydrodynamics becomes important, and various quasi-equilibrium approximations are used
  – the final merger requires evolution with full general relativity and hydrodynamics
Breakdown of Approximation

• At some small separation of the neutron stars, the assumptions behind a quasi-equilibrium approximation become invalid, and numerical simulation is necessary.

• It is necessary to determine in what range the quasi-equilibrium approximation is valid:
  – so that the quasi-equilibrium sequence can be used as initial data for the numerical simulation
  – due to computational resource limitations, we want to start with as small a separation as possible
CFQE Approximation Assumptions

– Conformal Flatness
  • The gravitational radiation timescale $\gg$ orbital timescale
  • gravitational waves can be ignored to first approximation
  • the spatial 3-metric $\gamma_{ij}$ is flat except for a local conformal factor $\psi$:
    $\gamma_{ij} = \psi^4 \delta_{ij}$

– Quasi-Equilibrium
  • the spacetime is essentially stationary in the corotating frame
  • the four velocity of the fluid depends on the spin (corotational or irrotational)
  • A CFQE sequence is constructed by piecing together constant baryonic mass CFQE configurations that evolves as the separation decreases due to the emission of gravitational radiation.
Starting numerical evolutions with CFQE?

• The “standard” is to start numerical evolutions using CFQE initial data with a separation near the ISCO, ~3R
  – is that configuration still valid as part of CFQE sequence?
  – If an evolution using a CFQE configuration as initial data deviates significantly from the CFQE-sequence spacetime on suborbital time scales, then that particular CFQE configuration cannot be a good starting point for numerical evolution.

• It has been shown that corotational CFQE is a poor approximation at close (<4.5R) separation (Miller, Gressman, Suen 2004)

• Is the situation better with Irrotational CFQE?
  Not, really
Measuring the Violation of Conformal Flatness

• We need a coordinate invariant way to measure the accuracy of the conformal flatness assumption:

\[ B_{ijk} = 2D[i \left( (3)R_j \right)_k - \frac{1}{4} \gamma_j]_k (3)R \]

• the Cotton-York tensor is then given by:

\[ H_{ij} = \epsilon^{mn} j B_{mni} \]

• the scalar H is the matrix norm of H_{ij}, normalized by the covariant derivative of the 3-Ricci tensor:

\[ H = \frac{|H_{mn}|}{\sqrt{D_i (3)R_j k D^i (3)R_j k}} \]

• for a global measure, we define the baryonic density weighted norm:

\[ \langle H \rangle_\rho = \frac{\int d^3x |H| \sqrt{\gamma} \rho W}{\int d^3x \sqrt{\gamma} \rho W} \]
Techniques

• We use Irrotational CFQE spectral data sets from the Meudon Group
  – Polytropic equation of state: $P = k \rho \Gamma$, $\Gamma = 2$
  – baryonic mass of a single star: $M_0 = 1.625 \, M_{\text{sun}}$

• Import the data into each of our Adaptive Mesh Refinement (AMR) grids as initial data

• Evolve the system from this initial data and track the violation of the conformal flatness
Irrotational vs. Corotational

Irrotational configuration:
\[ M_0 = 1.625 \, M_{\text{sun}} \]
separation = 18.76 \( M_0 \) (45 km)

Corotational configuration:
\[ M_0 = 1.49 \, M_{\text{sun}} \]
separation = 23.44 \( M_0 \) (51.55 km)
CFQE is violated in a fraction of an orbit
CF violation at different separations

Irrotational configurations:
\( M_0 = 1.625 \, M_{\text{Sun}} \)

Corotational configurations:
\( M_0 = 1.49 \, M_{\text{Sun}} \)

\( \Omega_1,2 = 18.76 \, M_0 \quad \Delta x = 0.3077 \, M_0 \)
\( \Omega_1,2 = 25.02 \, M_0 \quad \Delta x = 0.3077 \, M_0 \)
\( \Omega_1,2 = 29.18 \, M_0 \quad \Delta x = 0.3077 \, M_0 \)

(Miller, Gressman, Suen, 2004)
When is CFQE valid?

Irrotational configurations:
\[ M_0 = 1.625 \, M_{\odot} \]

Premilinary results

Corotational configurations:
\[ M_0 = 1.49 \, M_{\odot} \]
(Miller, Gressman, Suen, 2004)
Conclusions

- Irrotational CFQE sequences also show rapid violation of Conformal Flatness at close separations
- Future Work to do:
  - higher resolutions, determine error bars
  - coordinate invariant check of the Quasi-Equilibrium assumption
  - extend initial separations (to the pt that CFQE is valid)
  - check other quasi-equilibrium approximations