Simulations of rotating, magnetized core collapse in full general relativity

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Introduction

What causes supernovae to explode?

- Neutrino driven convection
  *Woosley & Janka, Nature Physics 1, 147 (2005).*

- Rotation effects (e.g. viscous heating)

- Magnetic effects (e.g. the MRI)

- Acoustic oscillations
Initial data

We start with a uniformly rotating, equilibrium star with

\[ P = K_0 \rho^\Gamma, \quad \Gamma = 4/3. \]

Choosing \( K_0 = 5 \times 10^{14} \) cgs gives:

\[ M = 1.503M_\odot, \quad R = 2267 \text{ km}, \quad T/|W| = 8.9 \times 10^{-3} \]
\[ P_c = 1.53 \text{s}, \quad R_p/R_{eq} = 0.667, \quad J/M^2 = 1.235 \]

Initial magnetic field:

\[ A_\varphi = A_b \omega^2 \max[\rho - \rho_{\text{cutoff}}, 0]. \]

\( A_b \) is chosen such that:

\[ E_{EM}/T = 1.2 \times 10^{-2}, \quad \max(P_{\text{mag}}/P_{\text{gas}}) = 1.3 \times 10^{-3}, \]
\[ \max(B^z) \sim 2 \times 10^{13} \text{G}. \]
Initial data, continued
Equation of state

We employ a hybrid EOS (Zwerger & Müller, A&A 320, 209, 1997):

\[ P(\rho, \epsilon) = P_{\text{cold}}(\rho) + P_{\text{th}}(\rho, \epsilon) \]

\[ P_{\text{cold}}(\rho) = \begin{cases} K_1 \rho \Gamma_1, & \rho \leq \rho_{\text{nuc}} \\ K_2 \rho \Gamma_2, & \rho \geq \rho_{\text{nuc}} \end{cases} \]

where \( K_2 = K_1 \rho_{\text{nuc}}^{\Gamma_1 - \Gamma_2} \). The thermal pressure comes from

\[ P_{\text{th}} = (\Gamma_{\text{th}} - 1) \rho \epsilon_{\text{th}}, \quad \epsilon_{\text{th}} = \epsilon - \epsilon_{\text{cold}} \]

We choose

\[ \Gamma_1 = 1.3 , \quad \Gamma_2 = 2.5 , \quad \Gamma_{\text{th}} = \Gamma_1 , \quad \rho_{\text{nuc}} = 2 \times 10^{14} \text{ g cm}^{-3} \]

\[ K_1 = K_0 \rho_0^{4/3 - \Gamma_1} \quad (\rho_0 = 1 \text{ g cm}^{-3}) \]
Equation of state, continued

To trigger the collapse, we set

\[ P_{\text{cold}} = K_1 \rho^{\Gamma_1}, \quad \varepsilon = \frac{P_{\text{original}}}{(\Gamma - 1) \rho}. \]

This leads to a 10% pressure depletion.
Results: Unmagnetized
Results: Unmagnetized
Results: Unmagnetized
Results: Unmagnetized

\[ t = 124.48 \text{ ms} \]
Results: Unmagnetized

\[ t = 150.38 \text{ ms} \]
Results: Magnetized

\[ t = 115.36 \text{ ms} \]
Results: Magnetized

\[ t = 115.96 \text{ ms} \]
Results: Magnetized
Results: Magnetized

\[ t = 124.49 \text{ ms} \]
Results: Magnetized

\[ t = 130.96 \text{ ms} \]
Results: Magnetized

$t = 139.14 \text{ ms}$
Results: Magnetized
Results: Magnetized

\[ t = 150.44 \text{ ms} \]
Results, continued

\[ \rho_c \text{ (g/cm}^3\text{)} \]

\[ \alpha_c \text{ (cm)} \]

\[ t \text{ (ms)} \]

unmagnetized
magnetized
Comparison of codes

- **Dotted line**: Duez, Liu, Shapiro, & Stephens PRD 72, 024028 (2005)
- **Solid line**: Shibata & Sekiguchi PRD 72, 044014 (2005)
Comparison of codes, continued

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MHD outflows

\[
M_{\text{outflows}} = \int_{r=\text{const}} dA \rho_{\star} v^r,
\]

\[
E_{\text{outflows}} = -\int_{r=\text{const}} dA \alpha \sqrt{\gamma} T^r_t,
\]

\[
J_{\text{outflows}} = \int_{r=\text{const}} dA \alpha \sqrt{\gamma} T^r_\varphi,
\]