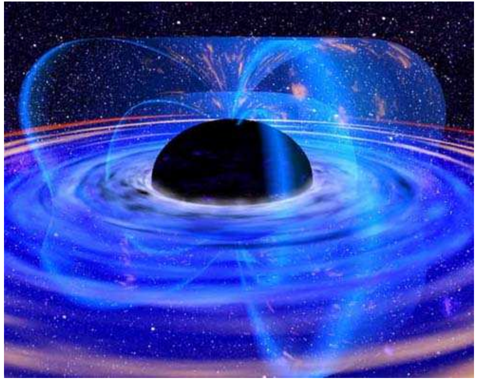

Modified Theories of Gravity
in
Cosmology

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About this talk . . .

- Motivation:

  General Relativity by itself seems unable to justify the late-time cosmic acceleration. **Corrections to the Einstein-Hilbert lagrangian relevant at very low cosmic curvatures** have been proposed as a mechanism for the late-time cosmic speed-up.

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- Aims:
  - Introduce some families of modified theories of gravity.
  - Show how cosmic speed-up arises in $f(R)$ models.
  - Discuss the influence of the cosmic dynamics on local systems.
  - Use elementary observational facts to constrain the form of the lagrangian $f(R)$.

Gravity lagrangians and fields

- Modified lagrangians are functionals of curvature invariants:

$$R = g^{\mu\nu} R_{\mu\nu} \qquad P = R^{\mu\nu} R_{\mu\nu} \qquad Q = R^{\alpha}_{\ \mu\beta\nu} R_{\alpha}^{\ \mu\beta\nu}$$

Popular models $\longrightarrow f(R)$ $\qquad$ $f(R,P,Q) \longleftarrow$ Not so popular

Examples:

$$f(R) = R + \frac{R^2}{M^2} \qquad f(R) = R - \frac{R_0^2}{R} \qquad f(R,P,Q) = R - \frac{R_0^3}{aR^2 + bP + cQ}$$

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- Dynamical fields:
  - Metric formalism: $\longrightarrow g_{\mu\nu}$ (fourth-order e.o.m.)
  - Palatini formalism: $\longrightarrow g_{\mu\nu}, \Gamma^{\alpha}_{\beta\gamma}$ (second-order e.o.m.)

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- By requiring **second-order equations and covariance** in metric formalism we are **uniquely** led to the EH lagrangian $R - 2\Lambda$.
- **There is no selection rule** for the lagrangian neither in Palatini nor in fourth-order theories. Could cosmology be a guide?

New dynamics at low R in $f(R)$ models

- The e.o.m. for $f(R) = R + \epsilon g(R)$ are : $(f' = df/dR, g' = dg/dR)$

$$G_{\mu\nu} = \frac{\kappa^2}{f'} T_{\mu\nu} + \frac{\epsilon}{f'} \left[g_{\mu\nu}(Rg'(R) - g(R)) - (\nabla_\mu \nabla_\nu g'(R) - g_{\mu\nu} \Box g'(R)) \right]$$

$\hookrightarrow$ trace $\rightarrow$ $R = -\kappa^2 T - \epsilon \left[3 \Box g'(R) + R g'(R) - 2g(R) \right]$

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- **Is the solar system affected by that change of regime?**
  - NO: because $R_{SS} \gg R_\varepsilon$ due to a cloud of dust and plasma.
  - Who knows: Since R is a dynamical object ...

$\Rightarrow$ Is the local R sensitive to the asymptotic boundary values?

The Solar System in $f(R)$ models

- Expanding about $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ we find:

$$h^{(2)}_{00} \approx 2G\frac{M_\odot}{r} + \frac{\Lambda_c}{6}r^2$$

$$h^{(2)}_{ij} \approx \delta_{ij}\left[2\gamma G\frac{M_\odot}{r} - \frac{\Lambda_c}{6}r^2\right]$$

with

$$M_\odot = \int d^3x \rho_{sun}, \quad \Lambda_c = \frac{R_c f'_c - f(R_c)}{f'_c}$$

$$G = \frac{\kappa^2}{8\pi f'(R_c)}\left[1 + \frac{e^{-m_c r}}{3}\right]$$

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where $m_c^2 \equiv \frac{f'(R_c) - R_c f''(R_c)}{3 f''(R_c)}$ is a function of R_c. $\begin{cases} f' = 1 + \epsilon g'(R) \\ f'' = 0 + \epsilon g''(R) \end{cases}$

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- **The change in the cosmic R_c leads to dramatic changes:**
  - For $R_c \gg R_\epsilon$ and $(f''(R_c) \to 0,\ m_c \to \infty)$ $\Leftrightarrow$ **GR regime** .
  - As $R_c \to R_\epsilon$ then $(f''(R_c) > 0,\ m_c \to$finite$)$ $\Leftrightarrow$ **New dynamics** .

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- The local R also changes with the expansion

$$R = \Lambda_c + \frac{m_c^2\kappa^2}{4\pi f'(R_c)}\int d^3x' \frac{\rho(t, \vec{x}')}{|\vec{x} - \vec{x}'|} e^{-m_c|\vec{x} - \vec{x}'|}$$

The $f(R)$ lagrangian according to experiments

- The cosmic expansion changes the effective mass

$$m_c^2 \equiv \frac{R_c}{3}\left[\frac{f'(R_c)}{R_c f''(R_c)} - 1\right]$$

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$$\left[\frac{f'(R)}{R f''(R)} - 1\right] \geq \frac{1}{l_0^2 R} \rightarrow \frac{d \ln f'}{dR} \leq \frac{l_0^2}{1+l_0^2 R} \rightarrow f(R) \leq A + B(R + \frac{l_0^2 R^2}{2})$$

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- Since $f' > 0$ and $f'' > 0$ it is also bounded from below:

$$-2\Lambda \leq f(R) \leq R - 2\Lambda + \frac{l_0^2 R^2}{2}$$

See G.J.O. *Phys.Rev.Lett.* **95**,261102 (2005),
and G.J.O. *Phys.Rev.*~**D72**,083505 (2005)

Summary and conclusions

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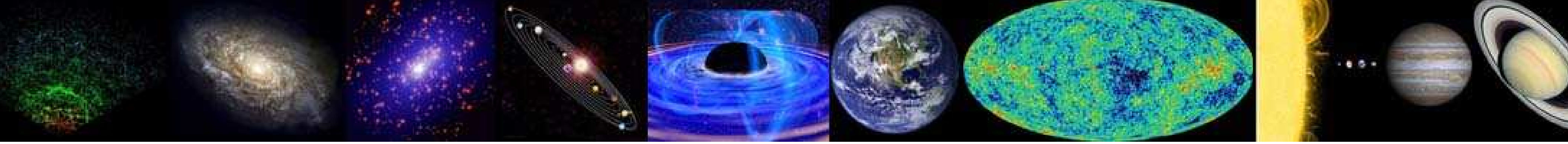
Moral

The dynamics of **local systems** in modified theories of gravity **might be very sensitive to the cosmic evolution via boundary conditions**. **In particular, such effects are likely to manifest in theories of the form** $f(R,P,Q)$ [Work in progress]



Thanks !!!