Scalar self-force for eccentric orbits in Schwarzschild spacetime

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Self force

- Inspiral of particle is driven by the regular part of the self-force:
  \[ m u^\alpha \nabla_\alpha u^\beta = q(g^{\alpha\beta} + u^\alpha u^\beta) \nabla_\alpha \Phi^R \]
  where \( \Phi^R = \Phi^{\text{ret}} - \Phi^S \)

- Find this correction by assuming geodesic motion
  \[ \Box \Phi^{\text{ret}} = -4\pi \rho \quad \rho(x) = q \int_\gamma \delta_4(x, z(\tau)) \, d\tau \]

- and regularize via a multipole-coefficient decomposition of tetrad components
  \[ \Phi^R_{(\mu)}(z) = \lim_{x \to z} \sum_l \left[ \Phi^{\text{ret}}_{(\mu)l}(x) - \Phi^S_{(\mu)l}(x) \right] \]
Wave equation

• $\Phi_{\text{ret}}$ is found by numerically solving

$$\Box \Phi = -4\pi q \int_\gamma \delta_4(x, z(\tau)) \, d\tau$$

via a spherical harmonic decomposition.

$$\Phi = \sum_{l,m} \frac{1}{r} \psi_{lm}(r, t) Y_{lm}(\theta, \phi)$$

• Reduced wave equation

$$\left\{ -\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial r^*^2} - f \left[ \frac{\ell(\ell + 1)}{r^2} + \frac{2M}{r^3} \right] \right\} \psi_{lm} = S_{lm}$$

$$S_{lm} = -\frac{4\pi f(r(t))}{r(t)E} \bar{Y}_{lm}(\frac{\pi}{2}, \varphi(t)) \delta(r^* - r^*(t))$$

$$f = 1 - \frac{2M}{r}, \quad r^* = r - 2M \ln\left(\frac{r}{2M} - 1\right)$$
Numerical method (I)

- Characteristic grid evolution in the time domain [Lousto & Price, PRD 56 6439, 1997], [Lousto, CQG 22 S543, 2005]

\[-4 \iint \partial_u \partial_v \psi_{lm} \, du \, dv - \iint V_i(r^*) \psi_{lm} \, du \, dv = \int \tilde{S}(\tau) \, d\tau\]

- Fourth-order accurate algorithm

- No boundary conditions are enforced, instead the grid matches the domain of dependence in each timestep

- No physical initial data is specified, we wait until the initial radiation contents has propagated away

- Both the field and the source term are evolved concurrently
Numerical method (II)

- Vacuum cells:
  - Need all red points to advance
  - Modified Lousto's algorithm to use only points within the light-cone

- Sourced cells:
  - Introduce two piecewise polynomial around the position of the particle linked by jump conditions
  - Iteratively solve the integral for the top grid-point using a second order initial guess

\[
\begin{align*}
& \text{t+h} & \text{t} & \text{t-h} & \text{t-2h} \\
& r^*-2h & r^* & r^*+2h \\
& r^*-3h & r^*-h & r^*+h & r^*+3h
\end{align*}
\]
Numerical method (III)

- Extraction of $\psi_{lm}$ at the particle via a second piecewise polynomial
- Jump conditions can be calculated analytically
- Extraction is fourth order accurate for $\psi_{lm}$ and third order for $\nabla \psi_{lm}$
Vacuum regression of order 4
\( l = 6, m = 4, \Delta_t = 0.25 \)
Field value at the position of the particle
l = 6, m = 4, p = 7, e = 0.3, Delta_t = 0.125
Sourced regression of order 4
\[ l = 6, \, m = 4, \, p = 7, \, e = 0.3, \, \Delta_t = 0.25 \]
Particle on an eccentric orbit $p=7.8001$, $e=0.9$

- whirl: 3 orbits at $r \approx 4.1M$
- zoom: moves out to $r \approx 78M$
$e = 0.9$, $p = 7.8001$, $\Delta t = 1/32$, $t = 2000$
$e = 0.9, \ p = 7.8001, \ \Delta_t = 1/32, \ t = 2150$

\[
\log_{10}(\Phi_t) = 0.9, \ p = 7.8001, \ \Delta_t = 1/32, \ t = 2150
\]
$e = 0.9, p = 7.8001, q = 0.01, \Delta_t = 1/32M, l_{\text{max}} = 14$
Conclusions

- Calculation of the regularized self-force in the time domain is feasible both in regard to accuracy and computational effort.

- Future goals
  - Electromagnetic and gravitational cases
  - Use self-force to correct the particle's motion