Outer Trapped Surfaces in Vaidya Spacetimes

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Outline

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Background

Outer trapped surface - compact smooth spacelike 2-surface s.t. the expansion, $\Theta$, of future directed outgoing null geodesics orthogonal to it is negative.

Apparent horizon - given a spacelike 3-surface, outer boundary of region containing outer trapped surfaces that lie in the 3-surface.

- Given energy conditions lie inside event horizon.
  (As a result no such surfaces in flat space)
**Quasi-local 3-surfaces**

**Apparent 3-horizon** - Given foliation of spacetime by spacelike 3-surfaces, this is the union of apparent horizons on the leaves of the foliation

- Can be discontinuous
- Slicing dependent

**Trapping horizon** (Hayward, 1994)

**Dynamical horizon** (Ashtekar and Krishnan, 2003)

- Extra conditions - may not always exist
- Not unique
**Question**

What is boundary, in spacetime, of region containing outer trapped surfaces?

- Unique
- Not slicing dependent

Eardley (1997)

- Conjectured boundary of region containing outer trapped surfaces is event horizon
- Gave some heuristic arguments
Vaidya spacetimes

Vaidya metric:

\[ ds^2 = -\left(1 - \frac{R(v)}{r}\right)dv^2 + 2dvdr + r^2d\Omega^2 \]

\(R(v)\) non-negative, non-decreasing, bounded function. Least upper bound \(R_0\). \(R(v) = 2M\) is Schwarzschild metric. Stress energy, \(T_{ab} = \frac{R'(v)}{8\pi r^2}dv_a dv_b\)

- Spherically symmetric spacetime describing collapse of null dust fluid
- 3-surface \(r = R(v)\). Inside it (at given \(v\)) 2-spheres are outer trapped. Outside it they are not
Spacetime diagram: Collapse of null dust shell

- r=0 singularity
- Flat Space
- Schwarzschild
- Event Horizon
- Collapsing Shell
- Is this point, part of an outer trapped surface?
Outer trapped surfaces in Vaidya spacetimes

Main result: In Vaidya spacetimes the event horizon is the boundary of the region containing outer trapped surfaces
Main idea

- In flat space no outer trapped surfaces
- 2-spheres in flat space are inner trapped

Idea: Turn surface inside out by closing in certain way
Main idea
Main idea
Main idea
Key ideas

- Can construct axisymmetric surface by curve at fixed $\phi$ translated by axial Killing field

- Curve to be translated can be single integral curve of vector field
At much greater $v$...

- $r=0$ singularity
- $r=R(v)$
- Event Horizon
- Null Dust

Flat Space
Flat Space

$r=0$

$r=0$ singularity

At much greater $v$

Null Dust

Spacelike 3–surface

Event Horizon

$r=R(v)$

Null Dust

Flat Space
Flat Space

$r=0$

$r=0$ singularity

At much greater $v$

Null Dust

Spacelike 3−surface

$r=R(v)$

Event Horizon

Flat Space

$\text{Spacelike 3−surface}$

Null Dust
Flat Space
r=0
r=0 singularity
At much greater v
r=R(v)
Null Dust
Spacelike 3−surface
Event Horizon
Spacelike 3−surface
Spacelike 3–surface
Summary

- In Vaidya spacetimes outer trapped surfaces extend to event horizon
- Trapped surfaces do not extend everywhere to event horizon
- Expectation that result is correct in general since Vaidya spacetimes capture essentials of collapse

See gr-qc/0611057 for details